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# PROPERTIES OF A MODEL FOR THE TURBULENT MIXING OF THE BOUNDARY BETWEEN ACCELERATED LIQUIDS DIFFERING IN DENSITY

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A model has been proposed [1] for the turbulent mixing of the interface between accelerated liquids differing in density, which provides solutions to various problems in analytic form. This enables one to examine the behavior of the solution in relation to the empirical constants in the model.

A more complicated model for turbulent mixing is considered here that has three parameters, and the role of the newly introduced parameter is examined. Solutions are constructed for variable acceleration given by power, step, and sinusoidal laws. It is found that the width of the mixing region can vary by up to a factor 2 in accordance with the constant in the model that characterizes the role of the inertial mechanism. A solution is obtained for the mixing of a thin layer, and the problem is referred to an integral for the case of finite thickness.

1. Model with Three Parameters. Two incompressible liquids differing in density are placed in an accelerated vessel, and the boundary between them is unstable if the acceleration is directed from the light liquid into the heavy one. This is the Rayleigh-Taylor instability. If the viscosity and surface tension are negligibly small, as occurs for high accelerations, the boundary is disrupted. One substance begins to mix with the other, and experiment shows [2] that the mixing is turbulent.

There are semiempirical models for the turbulent mixing. A very simple one with one constant was proposed in [3]. An extension of the model is given in [4, 5].

The following is a more complicated semiempirical model for turbulent mixing with three parameters for a case of two incompressible substances:

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial x} D \frac{\partial \rho}{\partial x}; \quad (1.1)$$

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$$\frac{d\rho v^2}{2dt} = \rho D g \frac{\partial \ln \rho}{\partial x} - \nu \frac{\rho v^3}{l} + \frac{4}{3}\alpha_2 \rho D \left(\frac{d \ln \rho}{dt}\right)^2 + \frac{5}{6}\rho v^2 \frac{d \ln \rho}{dt}; \quad (1.2)$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}, \ \bar{u} = -D\frac{\partial \ln \rho}{\partial x}; \quad (1.3)$$

$$D = lv; \quad (1.4)$$

$$l = \alpha L, \quad (1.5)$$

where ρ is the density of the mixture of the heavy liquid ($\rho_1$) and the light one ($\rho_2$), $v^2$ is the turbulent energy, g is the acceleration, which is dependent only on time, u is the speed of the mixture, which is defined by (1.3) for incompressible liquids [3], L is the effective width of the mixing region, which is determined below, and α, ν, and $\alpha_2$ are empirical constants.

We assume that the interface at the initial instant t = 0 coincides with the origin x = 0, with the light and heavy liquids disposed respectively to left and right (Fig. 1).

The above system of equations has been derived by successive averaging of the initial gasdynamic equations. The true values of the density ρ, velocity u, pressure p, and entropy are replaced by the mean values and fluctuations: $\rho = \bar{\rho} + \rho'$, $p = \bar{p} + p'$, $u_k = \bar{u}_k + u_k'$, $k = 1, 2, 3$ etc. In the averaging we neglect the third correlations and the products of the second ones. The conservation equation for the turbulent energy is constructed in the usual way, with Prandtl's hypothesis used for the closure:

$$\overline{u_k'\rho'} = -D\frac{\partial \bar{\rho}}{\partial x_k},$$

$$\overline{u_k'u_i'} = \frac{2}{3}v^2\delta_{ki} - \alpha_2 D\left[\left(\frac{\partial \bar{u}_k}{\partial x_i} + \frac{\partial \bar{u}_i}{\partial x_k}\right) + \frac{2d}{3}\frac{\ln \bar{\rho}}{dt}\delta_{ki}\right]$$

and the Landau assumption:

$$\overline{u_k'\frac{\partial p'}{\partial x_k}} = \nu\bar{\rho}\frac{v^3}{l}.$$

In (1.1)-(1.3) and subsequently, the bar on $\bar{\rho}$ is omitted.

We now examine the meanings of the returns in (1.2). The first term in the right is the main one, which generates the turbulent mixing, and it is proportional to the square root of the Brent frequency, which characterizes the growth rate of the short-wave perturbations, and it is taken as zero in the case of stable flow, for negative values.

The second term incorporates the dissipation of the turbulent energy. This establishes the turbulence damping law in the absence of generating sources.

The third and fourth terms appear as a result of successive application of Prandtl's hypothesis. These terms were absent in [1].

These equations agree with the model of [4] if the liquids are incompressible.

We now consider the role of the last two terms in (1.2). Here we use the approximate approach of [1], where it was proposed to assume that $\partial v/\partial x = 0$. This approximation distorts the solution at the mixing front somewhat, but it greatly simplifies the initial equations, and in some cases enables one to obtain a solution in analytic form.

In fact, in that case the coefficient D is dependent only on time, so after the substitution

$$d\tau = D dt \quad (1.6)$$

equation (1.1) amounts to the diffusion equation, whose solution is known:

$$\rho = \frac{\rho_1 + \rho_2}{2} + \frac{\rho_1 - \rho_2}{2}\Phi\left(\frac{x}{2\sqrt{\tau}}\right),$$

where Φ is the probability integral.

To construct the solution as a whole, it is necessary to determine the energy $v^2$ and integrate (1.6). By width L of the mixing region we understood the effective width, which is found from the dimensionless density

$$\delta = (\rho - \rho_2)/(\rho_1 - \rho_2) = (1/2)(1 + \Phi) \tag{1.7}$$

as the distance between the points at which $\delta$ = 0.1 and 0.9.

It follows from (1.7) that the mixing front defined in this way propagates symmetrically to right and left and that the mixing width is

$$L = 4\eta_0 \sqrt{\tau},\ \eta_0 = 0.906. \tag{1.8}$$

We now derive an equation for v. In (1.2) we transfer to the Euler coordinate. We average both parts of the equation over the mixing region and construct an equation for $\bar{v}$. Here and subsequently, the bar over v will be omitted:

$$\frac{dv^2}{2d\tau} + k\frac{v^2}{\tau} = \frac{0{,}4\,(n-1)}{\eta_0\,(n+1)}\frac{g}{\sqrt{\tau}}; \tag{1.9}$$

$$k = \frac{\nu}{16\eta_0^2\alpha^2} + \frac{0{,}4}{3\sqrt{2\pi}}\left(\frac{n-1}{n+1}\right)^2 - \frac{16\eta_0^2\alpha^2}{3\pi^2}\alpha_2\left(\frac{n-1}{n+1}\right)^4, \quad n = \rho_1/\rho_2. \tag{1.10}$$

The solution to (1.9) is

$$v^2 = \frac{0{,}8\,(n-1)}{\eta_0\,(n+1)}\tau^{-2k}\int_0^{\tau} g\tau^{2k-1/2}d\tau.$$

We use (1.6) and (1.8) to get an equation defining $\tau(t)$. In some cases the solution can be constructed in analytic form. For example, if the acceleration g is constant up to a certain instant $t_0$ ($g = g_0$) and is then zero, we have

$$L = \begin{cases} A\frac{n-1}{n+1}g_0t^2, & t \leqslant t_0, \\ A\frac{n-1}{n+1}g_0t_0^2\left[2(1+2k)\frac{t-t_0}{t_0}+1\right]^{1/(1+2k)}, & t \geqslant t_0, \end{cases} \tag{1.11}$$

where

$$A = \frac{6{,}4\alpha^2\eta_0^2}{1+4k}.$$

This solution can be used in examining the effects of the parameters: the constants $\alpha$, $\nu$, and $\alpha_2$.

Parameter k according to (1.10) is dependent on $\alpha$, $\nu$, and $\alpha_2$ as well as on $(n-1)/(n+1)$, which is called the Atwood number. With the acceleration cut out, k determines the damping of the turbulent mixing. Theoretical estimates [6] imply that the turbulence decays in accordance with a 2/7 law, i.e., the characteristic turbulent length $l$ is dependent on time:

$$l \sim t^{2/7}.$$

Then for small Atwood numbers we have

$$1/(1+2k_0) = 2/7,\ k_0 = \nu/(16\eta_0^2\alpha^2) = 5/4. \tag{1.12}$$

The constant $A_0$ is determined at the stage of mixing with constant acceleration, which is described by the upper equation in (1.11). This is taken from experiment as $A_0$ = 0.09 [2].

Note that the mixing width is linearly dependent on the Atwood number when the latter is small (n ~ 1), which agrees with the results of [3], where the logarithmic relation $\ln n \simeq 2(n-1)/(n+1)$ was given.

In the general case, the dependence on the Atwood number is more complicated. While the value of $\alpha_2$ is unimportant at small Atwood numbers, there is an effect in the general case. Figure 2 gives the dependence of A and k on $(n-1)/(n+1)$ as constructed for $\alpha_2$ =

0 and 10. The curves show that A and k remain approximately constant and are only slightly dependent on $\alpha_2$ for $(n-1)/(n+1) < 0.6$.

Then the third parameter $\alpha_2$ can be determined only in carefully formulated experiments, which may reveal these relationships. In what follows we put $\alpha_2 = 0$.

2. Variable Acceleration. The acceleration is often inconstant in experiments, and therefore one uses a characteristic readily measured: the displacement s, which is related to the acceleration by the obvious equation

$$s = \int \left( \int g dt \right) dt. \tag{2.1}$$

One of the main parameters A characterizes the time course of the mixing and is usually determined from (1.11) and (2.1):

$$L/2s = A(n-1)/(n+1). \tag{2.2}$$

For constant acceleration and constant n, the value of A is constant. However, this is not so for variable acceleration. We consider three cases of time-dependent acceleration below: 1) power-law variation, 2) stepped with a constant period, and 3) sinusoidal.

1) Let

$$g = g_0 t^m, \tag{2.3}$$

where the power m may be positive or negative. Apart from special cases, we assume that $m > -1$.

The displacement s as a function of the variable $\tau$ is calculated in the appendix. Here (1.8) defines L as a function of $\tau$. Then the relation corresponding to (2.2) but for the case of variable acceleration is readily derived:

$$\frac{L}{2s} = \frac{2(1+4k)(1+m)}{(1+4k)(2+m)+m} A \frac{n-1}{n+1}. \tag{2.4}$$

Clearly, for m = 0 (constant acceleration), the latter expression becomes (2.2). The cofactor occurring for m > 0 is greater than 1 but for m < 0 it is less than 1.

Therefore, if (2.2) is used in processing the experimental data, the constant increases with rising acceleration, whereas it falls with decreasing. With k = 5/4

$$\frac{L}{2s} = \frac{1+m}{1+7m/12} A \frac{n-1}{n+1}.$$

There are two limiting cases. If we assume that k is small, the factor in (2.4) is independent of m. If m is close to −1, the acceleration falls rapidly and the factor is small, and mixing hardly occurs.

2) Let the acceleration follow the law

$$g = \begin{cases} 2g_0, & 2(i-1)t_0 < t < (2i-1)t_0, \\ 0, & (2i-1)t_0 < t < 2it_0, \quad i = 1, 2, \ldots \end{cases} \tag{2.5}$$

Such an acceleration is obtained at the interface between two slightly compressible liquids at rest at the initial instant that are accelerated from the light-liquid side by a piston with a constant acceleration.

In this formulation, the solution is not self-modeling. However, an asymptotic state is set up at large times, which we derive. The interface as a whole is displaced as for a constant acceleration $g_0$. The problem is to estimate the change in (2.2).

If k is small, this means that parameter $\nu$ is small and the inertia plays a large part in (1.2). We put formally k = 0.

Equations (1.4)-(1.6) and (1.9) are integrated, and the solution for the width takes the form

$$L(k=0) = A_0 \frac{n-1}{n+1} 2s, \quad A_0 = A(k=0) = 6{,}4\alpha^2\eta_0^2. \tag{2.6}$$

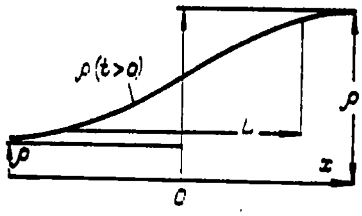


Fig. 1

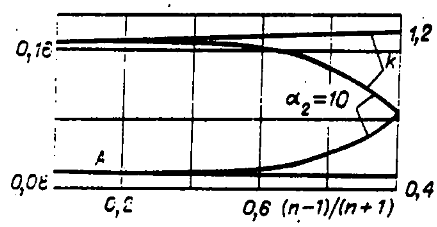


Fig. 2

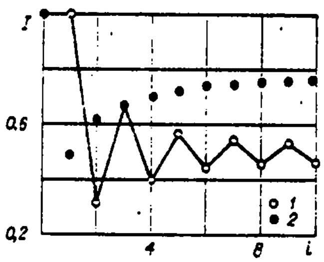


Fig. 3

This applies for an arbitrary time dependence of the acceleration.

If k is large (it is sufficient to take k >> 0.25), then (1.9) can be replaced by

$$v^2 = \frac{0.4\,(n-1)}{\eta_0\,(n+1)}\, g\, \frac{\sqrt{\tau}}{k},$$

and the solution is obtained in the form

$$L\,(k \gg 0{,}25) = A_1 \frac{n-1}{n+1}\left(\int_0^t \sqrt{g}\,dt\right)^2, \qquad (2.7)$$

$$A_1 = A\,(k \gg 0{,}25) = \frac{1{,}6\alpha^2\eta_0^2}{k},$$

which is equivalent to the solution in [3], where the inertia in the turbulent mixing was neglected.

For k small, (2.2) is also obeyed for variable acceleration on the basis of (2.6). If k >> 0.25, it follows from (2.7) that the ratio of (2.2) is no longer constant. In fact,

$$\frac{L\,(k \gg 0.25)}{2s} = A_1 \frac{n-1}{n+1} I\,(t),$$

$$I\,(t) = \left(\int_0^t \sqrt{g}\,dt\right)^2 \Big/ 2s.$$

The factor I(t) is dependent on time and can be calculated.

For the stepped acceleration of (2.5) we get $I(0) = 1$, while $I(\infty) = 0.5$ for large times; in fact, I(t) is given by

$$I\,(it_0) = \begin{cases} \dfrac{i+1}{2i}, & \text{if i is odd,} \\[2mm] \dfrac{i}{2\,(i+1)}, & \text{if i is even.} \end{cases} \qquad (2.8)$$

3) For sinusoidal acceleration

$$g = g_0(1 + \varphi \sin(\pi t/t_0))$$

the factor I is

$$I\,(it_0) = \frac{4i^2\,(1+\varphi)\,E^2\left(\frac{\pi}{2}, \sqrt{\frac{2\varphi}{1+\varphi}}\right)}{2\pi i\varphi + \pi^2 i^2},$$

where E is an elliptic integral of the second kind. If $\varphi = 1$, then $E(\pi/2, 1) = 1$ and

$$I(it_0) = 8i/(\pi^2 i + 2\pi). \tag{2.9}$$

Figure 3 shows the time dependence of I(t) (points 1 show (2.8) for stepped acceleration and points 2 show (2.9) for sinusoidal acceleration). In real models, $k \simeq 1.25$, so in that case I(t) will be close to the value obtained above for k >> 0.25.

3. Mixing of a Layer of Finite Width. A layer of material placed in a medium of a different density is displaced in any case, no matter what the sign of the acceleration, since one of the boundaries will always be unstable.

In the first stage, before the mixing region reaches the stable boundary, the solution will be self-modeling. Then the second stage begins, which is not self-modeling. An asymptotic solution for the second stage has been given [3] for one particular case.

The model of [1] enables one to refer the layer mixing to integrals, while the mixing of a thin layer can be considered analytically.

These limiting solutions are of essential value in checking the theory against experiment. According to section 1, the mixing of the heavy layer of thickness $x_0$ having density $\rho_1$ surrounded by material of density $\rho_2$ is described by the solution

$$\rho = \rho_2 + \frac{1}{2}(\rho_1 - \rho_2)\left[\Phi\left(\frac{x+x_0}{2\sqrt{\tau}}\right) - \Phi\left(\frac{x-x_0}{2\sqrt{\tau}}\right)\right].$$

Here for definiteness it has been assumed that the mixing begins at the boundary $x = x_0$ and the point $x = 0$ corresponds to the stable boundary.

We introduce the dimensionless density $\delta$ as follows:

$$\delta = \frac{\rho - \rho_2}{\rho(0, \tau) - \rho_2} = \frac{\Phi\left(\frac{x+x_0}{2\sqrt{\tau}}\right) - \Phi\left(\frac{x-x_0}{2\sqrt{\tau}}\right)}{2\Phi\left(\frac{x_0}{2\sqrt{\tau}}\right)}$$

The value of $\delta$ alters monotonically from 1 and $x = 0$ to 0 and $x = \infty$. There is no front, so as previously we introduced the effective thickness of the mixing region. We put correspondingly $x_{0.9} = x(\delta = 0.9)$, $x_{0.1} = x(\delta = 0.1)$. The values of $x_{0.9}$ and $x_{0.1}$ are found as the solutions to

$$\begin{aligned} \Phi\left(\frac{x_{0.9}+x_0}{2\sqrt{\tau}}\right) - \Phi\left(\frac{x_{0.9}-x_0}{2\sqrt{\tau}}\right) &= 1.8\Phi\left(\frac{x_0}{2\sqrt{\tau}}\right) \\ \Phi\left(\frac{x_{0.1}+x_0}{2\sqrt{\tau}}\right) - \Phi\left(\frac{x_{0.1}-x_0}{2\sqrt{\tau}}\right) &= 0.2\Phi\left(\frac{x_0}{2\sqrt{\tau}}\right). \end{aligned} \tag{3.1}$$

The effective width L is a certain function of $\tau$:

$$L = x_{0.1} - x_{0.9} = L(\tau). \tag{3.2}$$

We obtain an equation for the turbulent velocity averaged over the mixing region $\bar{v}$. As in section 1, the initial equation (1.2) is replaced by an approximate one. We put $\alpha_2 = 0$ and $g = g_0$, and get finally that

$$\frac{1}{2}\frac{dv^2}{d\tau} + \frac{\nu v^2}{\alpha^2 L^2} = g_0 \frac{\rho_{0.9} - \rho_{0.1}}{M} - \frac{v^2(\rho_{0.9} - \rho_{0.1})^2}{3\rho_{0.5} M L} \tag{3.3}$$

where $M \simeq (\rho_2 + \rho(0, \tau))/2L = \rho_{0.5} L$;

$$\rho_{0.5} - \rho_{0.1} = 0.8(\rho_1 - \rho_2)\Phi(x_0/2\sqrt{\tau})$$
$$\rho_{0.5} = \rho_2 + [(\rho_1 - \rho_2)/2]\Phi(x_0/2\sqrt{\tau}).$$

A difference from section 1 is that the integral $\int\limits_L \frac{\partial\rho}{\partial\tau}\,dx$ in the present case is different from zero. However, throughout the region $0 < x < \infty$ it is equal to zero, while in the mixing region L it is small, and therefore it is neglected.

Therefore, the solution is found by integrating the ordinary differential equation (3.3) together with the functional dependence of (3.2) for L as defined by the solution to (3.1). Equation (3.3) is linear in $v^2$, so the solution can be obtained as an integral. We use (1.6) to convert to the initial variable t:

$$t = \alpha \int\limits_0^\tau (1/vL)\,d\tau.$$

We now find the solution for the thin layer. This is found in analytic form. We consider times for which $\tau \gg x_0$. We use an approximate representation for the probability integral and expansion as a series to get the solution to (3.1) in explicit form:

$$x_{0.9} = 0.92\sqrt{\tau},\ x_{0.1} = 4.29\sqrt{\tau}.$$

From (3.2) we have an expression for L:

$$L = 3.37\sqrt{\tau}.$$

Then (3.3) becomes

$$\frac{1}{2}\frac{dv^2}{d\tau} + \frac{\nu v^2}{\alpha^2 (3.37)^2 \tau} = \frac{0.8 g_0}{3.37\sqrt{2\pi}}\,\frac{(\rho_1 - \rho_2)\,x_0}{\rho_2 \tau}. \tag{3.4}$$

Here we have neglected the $1/\tau^2$ term.

It follows from (3.4) that at large times v for a thin layer tends to a constant value $v_0$:

$$v_0 = \alpha\sqrt{1.35 x_0 g_0 (\rho_1 - \rho_2)/\pi\rho_2}, \tag{3.5}$$

and the width follows a linear law

$$L = 5.68\alpha v_0 t.$$

If the light layer of density $\rho_2$ is surrounded by the heavy one of density $\rho_1$, then $\rho_2$ in the denominator in (3.5) should be replaced by $\rho_1$.

We consider the limiting case where $\rho_2 = 0$. If the light liquid is surrounded by the heavy one, the turbulent mixing occurs at the maximum rate, and the width in the limit follows a linear law of maximal slope:

$$L = 3.72\alpha^2 (\sqrt{x_0 g/\nu})\,t. \tag{3.6}$$

If the heavy one is surrounded by the light one (vacuum in the limit), then (3.5) is not applicable. The equation for the turbulent velocity will be different, since in that case the mass of the mixed material is here bounded, and the equation for the turbulent velocity takes the form

$$\frac{1}{2}\frac{dv^2}{d\tau} + \frac{\nu v^2}{\alpha^2 (3.37)^2 \tau} = \frac{1.6 g_0}{3.37\sqrt{\tau}}.$$

We get a quadratic law for the width:

$$L = \frac{12.9\alpha^2 g}{2.84 + \nu/\alpha^2}\,t^2. \tag{3.7}$$

Note that the characteristics of the initial layer do not appear in the coefficient.

Within the framework of the above model, the asympototic laws of (3.6) and (3.7) are determined by the constants $\alpha$ and $\nu$, which are taken from a comparison with experiment for constant and zero accelerations. Only experiment can determine whether this is so in fact.

Appendix. We calculate s as a function of the variable τ. For this purpose we convert to τ in (2.3):

$$g = g_1 \tau^{m_1},$$

where $g_1$ and $m_1$ are at present unknown. From (1.2) we have

$$v^2 = \frac{0,8\,(n-2)}{\eta_0\,(n+1)} \frac{2g_1}{(4k+1+2m_1)} \tau^{(2m_1+1)/2}.$$

Substitution for v in (1.6) gives a relation between t and τ:

$$t = \sqrt{\frac{(n+1)(4k+1+2m_1)}{1.6\,(n-1) g_1 \eta_0}} \frac{\tau^{(1-2m_1)/4}}{(1-2m_1)\,\alpha}.$$

We use the identity

$$g_0 t^m = g_1 t^{m_1}$$

and find $m_1 = m/2(m+2)$,

$$g_1 = g_0^{\frac{2}{2+m}} \left[\frac{2\,(m+2)}{\alpha\,(m+4)}\right]^{\frac{2m}{2+m}} \left[\frac{(n+1)(2km+4k+m+1)}{0,8\,(n-1)\,\eta_0\,(m+2)}\right]^{\frac{m}{m+2}}.$$

We transform to τ in (2.1) to get

$$s = \frac{(4k+1)(m+2)+m}{(4k+1)(m+1)} \frac{\eta_0}{A} \frac{n+1}{n-1} \sqrt{\tau}.$$

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